e-ISSN: 3107-619X

NBPA Journal for Arts, Humanities & Social Sciences

A Peer-reviewed Multidisciplinary Quarterly Journal

Exploring the Uncertain Nature of Language in Relation to the Neutrosophic Set Theory

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Original Article

Abstract

Linguistic concepts are inherently imprecise and often lack well-defined boundaries, which limits the applicability of traditional theoretical frameworks. Fuzzy sets and fuzzy logic provide effective tools for modelling vagueness and partial membership; however, they are insufficient for representing all forms of uncertainty. To overcome these limitations, interval-valued fuzzy sets were developed to enhance uncertainty modelling. Furthermore, neutrosophic sets extend these approaches by offering a comprehensive framework capable of handling incomplete, indeterminate, and inconsistent information commonly encountered in real-world linguistic analysis.

Open Access

Received: 16.08.2026
Reviewed:01.09.2026
Accepted: 01.09.2026
Publication Date: July-September 2026
Volume: 2
Issue: 3
DOI: https://doi.org/10.65842/nbpa.v2.i3.006

Published by: North Bengal Philosophers Association
Website: https://nbpajournal.com

Keywords: Uncertainty, Impreciseness, Fuzzy logic, Intuitionistic logic, Neutrosophic logic.

Introduction:

Among the various paradigmatic changes in linguistics, one most crucial change concerns the concept of vagueness. The change has been manifested by a gradual transition from the traditional view to the modern view. The traditional view has defined the concept of vagueness in such a way as to dichotomise the individuals in some given universe of discourse into two groups: members and non-members. In this view, a very clear distinction exists between the membership and non-membership of the set. Though there are many concepts in natural language that do not have precise boundaries, such as tall people, intelligent people, close distance, more profit, etc. These concepts demand a gradual transition from membership to non-membership or vice versa. In this regard, a fuzzy set is defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Thus, an individual may belong to the fuzzy set to a greater or lesser degree, as indicated by a larger or smaller membership grade, as real number values ranging in the closed interval between 0 and 1"[1](Klir and Yuan, 1997).

On the other hand, Neutrosophic set is totally different from the classical binary set, multi-valued set or fuzzy set, intuitionistic set or vague set. Actually neutrosophic set is an extended form of fuzzy set or intuitionistic set. A fuzzy set allows only membership degrees between 0 and 1. But a neutrosophic set allows the membership degree outside the box, i.e. [0,1]. It may allow the membership degree to be more than 1 or less than 0. On the basis of this kind of membership degree that goes out of the box, there are three different neutrosophic sets: Neutrosophic Overset, Neutrosophic Underset and Neutrosophic Offset[2] (Smarandache, 2016).

In the domain of Neutrosophic discussion, some scientific notions have been introduced: 'over', 'under' and 'off' as prefixes to some verbs or predicates to designate the membership degree, which allows membership degrees more than 1, less than 0 and both less than 0 and more than 1. 'Overknowing' is like knowing someone or something over the line of knowledge. It is similar to overthinking, overconfidence, overproductive, etc. Similarly, 'Underknowing' is like knowing someone or something under the line of knowledge. It is similar to underthinking, underconfidence, underproductive, etc. 'Offknowing' is like knowing someone or something over or under the line of knowledge. It is similar to offconfidence, offproductive, offside, etc. In this connection, considering membership degree, 'overmembership' is just over the line of membership. If the line of membership lies between 0 and 1, 'overmembership' will be more than 1, 'undermembership' will be less than 0 and 'offmembership' will be more than 1 or less than 0[3](Smarandache, 2016).

Here, this new idea of membership degree allows degrees greater than 1 and less than 0, which is a clear contradiction of the classical membership degree [0,1]. So, this idea of membership degree does not fit the classical concept of membership degree, which allows only membership degrees within the closed interval between 0 and 1, i.e. [0,1].

Example: Let A be a set for an element x; the membership degree with respect to the set A may be greater than 1. For example, if John knows 6 characteristics of Alma, then John will be considered to know Alma well, and his membership degree with respect to knowing Alma well is 1 or T.

Also, there may be a case that Smith knows characteristics of Alma. Now, a question arises: what would be the membership degree of Smith with respect to knowing Alma well? Smith's membership degree may not be 1, because 1 corresponds to knowing 6 characteristics, and Smith knows 7 characteristics, which is more than membership degree 1. Here, Smith's membership degree may be considered as 'overknowing'.

Also, in the same methodology, if Jones knows no characteristics, which is considered to be membership degree 0 and also Jones knows something that does not characterise Alma. If it is the case, then again the same question arises: what would be the membership degree of Jones with respect to knowing Alma well?

If knowing no characteristics of Alma is considered to be a 0 membership degree, then the membership degree of Jones with respect to the same should be less than 0. It may be considered as 'underknowing'.

In this connection, now let us define neutrosophic overelement, underelement and offelement with respect to the neutrosophic components T, I and F:

"A neutrosophic overelement is an element which has at least one of its neutrosophic components T, I, F that is more than 1."A neutrosophic underelement is an element which has at least one of its neutrosophic components T, I, F that is less than 0. "A neutrosophic offelement is an element which has at least two of its neutrosophic components T, I, F such that one is more than 1 and one is less than 0"[4](Smarandache, 2016).

Neutrosophic Overset*:*

Neutrosophic overset is the set where one of the neutrosophic components (T, F, I) is more than 1.

Example: Let A be the set of 'believers in God'. Now, if 'believing in God corresponds to knowing the definitions of God, which designates the membership value 1, in that case, if someone knows the definition of God and also gives the justification in support of the definition, then the membership value of that person in the set A will be more than 1.

Neutrosophic Underset*:*

Neutrosophic underset is the set where one of the neutrosophic components (T, F, I) is less than

0.

Example: Let A be the set of 'believers in God'. Now, if 'do not believing in God corresponds to not knowing the definitions of God, which designates the membership value 0, in that case, if someone does not know the definition of God and also argues against the justification in support of the definition, then the membership value of that person in the set A will be less than 0.

Neutrosophic Offset:

Neutrosophic offset is the set where one of the neutrosophic components (T,F,I) is more than 1 and one is less than 0. Neutrosophic Offset is both Neutrosophic overset and Neutrosophic underset.

Example: let A be the set of 'believers in God'. Now if 'believing in God corresponds to knowing the definitions of God' which designates the membership value 1 and 'do not believing in God corresponds to not knowing the definitions of God' which designates the membership value 0, in that case if someone knows the definition of God and also gives the justification in support of the definition, then the membership value of that person in the set A will be more than 1 and also if he does not know the definition of God and also he argues against the justification in support of the definition, then the membership value of that person in the set A will be less than 0.

The methodology of the neutrosophic set is based on its three components, T, I, and F, and these components altogether are summed up to the value 1, though this summation may be extended to the value 3. These three components may sum up to the value 1 in classical fuzzy sets, fuzzy logic, classical fuzzy intuitionistic sets, and fuzzy intuitionistic logic. But in the neutrosophic set and neutrosophic logic, these three components independently provide information about the membership degree of an element with respect to a set. So the summation of these three components may exceed 1 and extend up to 3. This idea of neutrosophic set and logic goes beyond the upper bound of classical fuzzy intuitionistic set and logic, which is 1. Clearly, neutrosophic values are permitted to go outside the interval between 0 and 1.

Conclusion:

The concept of fuzzy sets was introduced by L. A. Zadeh in 1965. Since then, fuzzy sets and fuzzy logic have been widely used in many applications involving uncertainty. However, it has been observed that certain situations cannot be adequately addressed by fuzzy sets alone. To overcome this limitation, the concept of interval-valued fuzzy sets was introduced by L. A. Zadeh in 1975. Although fuzzy set theory has proven highly successful in handling uncertainties arising from vagueness or partial membership of an element in a set, it cannot model all types of uncertainties encountered in real-world problems, particularly those involving incomplete information. A further generalisation of fuzzy sets was proposed by K. T. Atanassov in 1986, known as intuitionistic fuzzy sets (IFSs).

In intuitionistic fuzzy sets, each element is characterised not only by a membership grade but also by a non-membership grade. Moreover, these two grades are subject to the condition that their sum is less than or equal to one. The concept of intuitionistic fuzzy sets can be regarded as an appropriate alternative when the available information is insufficient to describe imprecision using conventional fuzzy sets. Subsequently, intuitionistic fuzzy sets were extended to interval-valued intuitionistic fuzzy sets by K. T. Atanassov and G. Gargov in 1989.

Neutrosophic sets, proposed by Florentin Smarandache in 1998, 1999, 2002, 2005, 2006, and 2010, represent a further generalisation of fuzzy sets and intuitionistic fuzzy sets. They provide a powerful framework for dealing with incomplete, indeterminate, and inconsistent information present in real-world situations. Neutrosophic sets are characterised by three independent functions: the truth-membership function (T), the indeterminacy-membership function (I), and the falsity-membership function (F).

This theory has become significant in many application domains because it explicitly quantifies indeterminacy, and the truth-, indeterminacy-, and falsity-membership functions are mutually independent. In 2010, H. Wang, Florentin Smarandache, Y. Zhang, and R. Sunderraman introduced the concept of the single-valued neutrosophic set. A single-valued neutrosophic set independently expresses the degrees of truth-membership, indeterminacy-membership, and falsity-membership and effectively handles incomplete, indeterminate, and inconsistent information. The components of the single-valued neutrosophic set align well with human reasoning, reflecting the imperfect and limited knowledge that individuals acquire or observe from the external world.

References:

Klir, G. J., & Yuan, B. (1995). *Fuzzy sets and fuzzy logic: Theory and applications*. Prentice Hall.

Smarandache, F. (2016). *Neutrosophic overset, neutrosophic underset, and neutrosophic offset: Similarly for neutrosophic over-/under-/off-logic, probability, and statistics*. Pons Editions.

Smarandache, F. (n.d.). *Neutrosophy*. University of New Mexico. Neutrosophy – University of New Mexico

Smarandache, F. (Ed.). (n.d.). *Neutrosophic Sets and Systems*. University of New Mexico. Neutrosophic Sets and Systems

Smarandache, F. (2016). Degree of dependence and independence of the (sub)components of fuzzy set and neutrosophic set. *Neutrosophic Sets and Systems, 11*, 95–97.

Smarandache, F. (2016). *Vietnam veteran în științe neutrosofice: Instantaneous photo-video diary*. Editura Mingir.

Smarandache, F. (2016). *Neutrosophic overset applied in physics* [Conference presentation]. 69th Annual Gaseous Electronics Conference, Bochum, Germany, October 10–14, 2016.

Popescu, D. P. (2016, June 21). Să nu ne sfiim să gândim diferit—De vorbă cu prof. univ. dr. Florentin Smarandache. *Observatorul*. Observatorul article

Smarandache, F. (2015). *Symbolic neutrosophic theory*. EuropaNova. Symbolic Neutrosophic Theory

Smarandache, F. (2003). *Introduction to neutrosophic measure, neutrosophic integral, and neutrosophic probability*

Smarandache, F. (2014). *Introduction to neutrosophic statistics*. Sitech. Introduction to Neutrosophic Statistics .

Smarandache, F. (2016, May 18). *Foundations of neutrosophic logic and set and their applications to information fusion* [Tutorial]. Hanoi University, Hanoi, Vietnam.

Smarandache, F. (2016, May 17). *Neutrosophic theory and applications* [Lecture]. Le Quy Don Technical University, Faculty of Information Technology, Hanoi, Vietnam.

Smarandache, F. (2015, June 6). *Types of neutrosophic graphs and neutrosophic algebraic structures together with their applications in technology* [Conference presentation]. Universitatea Transilvania din Brașov, Faculty of Product Design and Environment, Brașov, Romania.

Smarandache, F. (2014, July 7). *Foundations of neutrosophic logic and set and their applications to information fusion* [Tutorial]. 17th International Conference on Information Fusion, Salamanca, Spain.